

B_s Decays and Mixing

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Theoretical remarks are offered regarding recent hadron collider results on the mixing and decays of B_s mesons. Topics covered include: (1) CP-violating mixing in $B_s(\overline{B}_s) \rightarrow J/\psi\phi$, (2) the D0 dimuon charge asymmetry, (3) information from triple products, (4) $B_s \rightarrow J/\psi f_0$, (5) new physics constraints, (6) some illustrative new physics scenarios.

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1. Introduction

Recent results on B_s decays and mixing have been presented by the CDF and D0 Collaborations at the Fermilab Tevatron and the LHCb Collaboration at CERN. We begin by discussing CP-violating mixing in B_s (\bar{B}_s) $\rightarrow J/\psi\phi$. Experiments at CDF and D0 suggested a mixing phase β_s much larger than that in the Standard Model (SM). With such a large phase, we pointed out that time-dependent decays should display explicit time-dependence [1]. We update that analysis in Section 2.

The D0 Collaboration has presented evidence for a charge asymmetry in same-sign dimuons produced in $\bar{p}p$ collisions at $\sqrt{s} = 1.96$ TeV [2]. We suggest in Section 3 a test of whether this asymmetry is due to decays of b quarks, as claimed, or background sources such as kaons [3].

In Section 4 we discuss what triple products in $B_{(s)} \rightarrow V_1 V_2$ actually measure. The answer [4] is CP violation, but only under certain conditions. The study of $B_s \rightarrow J/\psi f_0$, mentioned in Section 5, avoids the angular analysis needed to interpret $B_s \rightarrow J/\psi\phi$. In Section 6, we note constraints on new physics, and comment in Section 7 on a couple of scenarios for consideration should any hints for physics beyond the SM be borne out by further tests. We conclude in Section 8.

2. CP violation in interference between B_s - \bar{B}_s mixing and $B_s \rightarrow J/\psi\phi$ decay

For formalism we refer to [5]. B_s - \bar{B}_s mixing is expected to be dominated by the top quark in box graphs. The observed values $\Delta m_s = (17.77 \pm 0.10 \pm 0.07)$ ps⁻¹ (CDF [6]) and $(17.63 \pm 0.11 \pm 0.04)$ ps⁻¹ (LHCb [7]) agree with SM predictions. Denoting

$$|B_{sL}\rangle = p|B_s\rangle + q|\bar{B}_s\rangle ; \quad |B_{sH}\rangle = p|B_s\rangle - q|\bar{B}_s\rangle , \quad (2.1)$$

we expect for $\Delta\Gamma \ll \Delta m$, $q/p \simeq \exp(2i\beta_s)$, $\beta_s^{\text{SM}} = -\text{Arg}(-V_{ts}^* V_{tb}/V_{cs}^* V_{cb}) = (1.04 \pm 0.05)^\circ$ [5]. The SM $B_s \rightarrow J/\psi\phi$ CP asymmetry then should be governed by the small mixing phase $\phi_M = -2\beta_s^{\text{SM}}$.

In 2008, CDF [8] and D0 [9] favored a mixing phase differing from $-2\beta_s$ by $\sim 2.2\sigma$ based on the decay $B_s \rightarrow J/\psi\phi$. At that time we pointed out that such a large mixing phase (the illustrative value was then $\phi_M = -44^\circ$ [8]) would imply detectable time-dependence of angular distribution coefficients, differing for tagged B_s and \bar{B}_s [1].

We review the discussion briefly. For a CP test, one tags the flavor at $t = 0$, denoting $\eta = \pm 1$ for a tagged (B_s, \bar{B}_s) . The coefficients of helicity amplitudes $|A_\parallel|^2$, $|A_\perp|^2$ describing different angular dependences are denoted by \mathcal{T}_+ , \mathcal{T}_- , where

$$\mathcal{T}_\pm \equiv e^{-\Gamma t} [\cosh(\Delta\Gamma t)/2 \mp \cos(\phi_M) \sinh(\Delta\Gamma t)/2 \pm \eta \sin(\phi_M) \sin(\Delta m_s t)] . \quad (2.2)$$

Taking $\phi_M = -44^\circ$, $\Delta\Gamma/\Gamma = 0.228$, and assuming the tagging η to be diluted by a factor of 0.11, we concluded that wiggles should be distinguishable between the B_s -tagged and \bar{B}_s -tagged \mathcal{T}_\pm distributions. We advocated making such a plot as evidence for CP violation in $B_s \rightarrow J/\psi\phi$ at a level beyond the SM. Here we update our estimate of t -dependence, finding the oscillations a bit smaller, but still visible. We take $\phi_M = (-39 \pm 17)^\circ$ based on an average between CDF [10, 11] and D0 [12] values, choose $\Delta\Gamma/\Gamma = 0.143$ based on an average between CDF ($0.075 \pm 0.035 \pm 0.010$) and D0 ($0.15 \pm 0.06 \pm 0.01$), and continue to assume a dilution factor of 11%. The resulting plot is shown in Fig. 1.

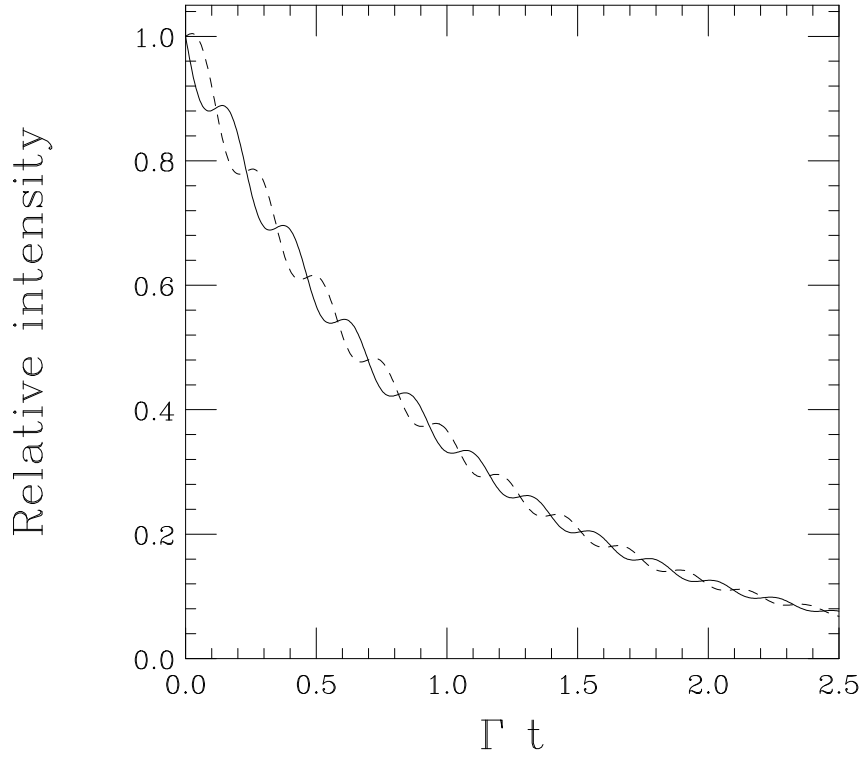


Figure 1: Relative intensities of \mathcal{T}_+ signals as functions of Γt , for B_s tags (solid) and \bar{B}_s tags (dashed). This figure represents an update of a similar one in Ref. [1].

At this Conference, LHCb presented data restricting ϕ_M to the range $[-2.7, -0.5]$ [13] (68% c.l.), 1.2σ from the SM. We are eagerly awaiting data from ATLAS and CMS.

3. D0 dimuon asymmetry – Is it due to b 's? K 's?

The SM predicts a small asymmetry in the yield of same-sign muon pairs due to $b\bar{b}$ production followed by meson \Leftrightarrow antimeson oscillation: $A_{sl}^b \equiv \frac{N^{++} - N^{--}}{N^{++} + N^{--}} = (-2.0 \pm 0.3) \times 10^{-4}$ [14]. The D0 Collaboration reports a much larger value, $A_{sl}^b = (-9.57 \pm 2.51 \pm 1.46) \times 10^{-3}$, nearly 50 times the SM value [2]. (CDF is not ready to report such a measurement but has quoted a new average mixing parameter $\bar{\chi}$ [15].)

D0 has interpreted its result as 3.2σ evidence for CP violation in neutral B mixing. They have performed 16 systematic checks for which their results are found consistent with their nominal ones. Estimating the correct kaon decay backgrounds is crucial.

We have suggested a test [3] to see if a smaller asymmetry is obtained in a sample depleted in $b\bar{b}$ pairs. If one reduces the maximum allowed impact parameter of muon tracks, the signal should vanish more rapidly than background. The effect of our suggestion, an impact parameter cut of $b < 100\mu\text{m}$, is not yet known to us.

We denote quantities in the B rest frame with an asterisk (*) and those in the lab frame with none. The lab energy of the B is $E_B = \gamma m_B = m_B / \sqrt{1 - \beta^2}$. Muon angles with respect to the B

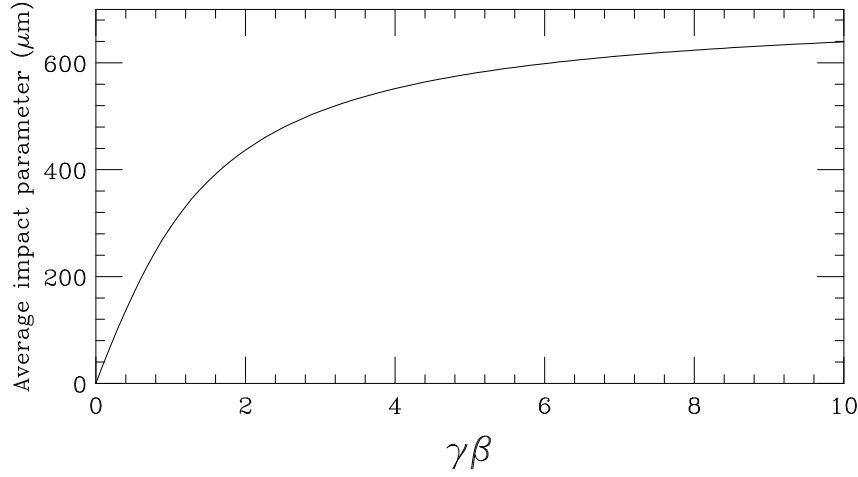


Figure 2: Dependence of $\langle b \rangle$ on $\gamma\beta$ [3].

Table 1: Fraction of events remaining for a given $\langle b \rangle$ when events with $b > b_0$ are discarded [3].

| b_0 (μm) | 100 | 200 | 300 | 400 | 500 |
|---------------------------------------|-------|-------|-------|-------|-------|
| $\langle b \rangle$ (μm) | | | | | |
| 150 | 0.237 | 0.542 | 0.748 | 0.866 | 0.930 |
| 300 | 0.080 | 0.237 | 0.400 | 0.542 | 0.658 |
| 450 | 0.040 | 0.129 | 0.237 | 0.347 | 0.450 |

boost are denoted by θ^* in the B rest frame and θ in the lab. The transformation between them is $\sin \theta = \sin \theta^* / [\gamma(1 + \beta \cos \theta^*)]$. The isotropy of muon emission in $\cos \theta^*$ can be used to calculate the average values of $\sin \theta$ and $b = \gamma\beta \sin \theta c\tau$, where $c\tau = 450\mu\text{m}$ and

$$\langle \sin \theta \rangle = \frac{1}{2} \int_0^\pi \frac{\sin^2 \theta^* d\theta^*}{\gamma(1 + \beta \cos \theta^*)} = \frac{\pi}{2} \frac{1}{1 + \gamma}. \quad (3.1)$$

The dependence of $\langle b \rangle$ on $\gamma\beta$ is shown in Fig. 2.

An eyeball fit to the CDF b distribution [16] gives $\langle b \rangle = 350 \mu\text{m}$. Table 1 denotes the effect of discarding events with b exceeding various values of b_0 .

The D0 Collaboration defines a transverse impact parameter b_\perp relative to the closest primary vertex and a longitudinal distance b_\parallel from the point of closest approach to this vertex. They choose $b_\perp < 3000 \mu\text{m}$ and $b_\parallel < 5000 \mu\text{m}$. These are related to b as follows. The transverse and longitudinal components of muon momentum in the lab are $p_\perp^\mu = p^\mu \sin \psi$, $p_\parallel^\mu = p^\mu \cos \psi$. The distance d of a point along the μ trajectory from the vertex is $d^2 = b_\perp^2 + (s \sin \psi)^2 + (s \cos \psi - b_\parallel)^2$, where s is the distance along the μ trajectory from the transverse point of closest approach. The minimum of d is $b = d_{\min} = [b_\perp^2 + (b_\parallel \sin \psi)^2]^{1/2}$. Little signal reduction is seen with $b_\perp < 500 \mu\text{m}$, $b_\parallel < 500 \mu\text{m}$ [2], but we advocate a tighter cut. The key question remains with regard to D0 muons:

are they really from b decays? This question should be answered by imposing an upper bound of $b_0 < 100\mu\text{m}$ on the impact parameter b_0 .

4. What do triple products in $B_{(s)} \rightarrow V_1 V_2$ measure?

A spinless particle decaying to four spinless particles gives rise to three independent momenta in its rest frame. One can form a T-odd expectation value out of (e.g.) $\mathbf{p}_1 \times \mathbf{p}_2 \cdot \mathbf{p}_3$ [4, 17]. A famous example is the asymmetry of $(13.6 \pm 1.4 \pm 1.5)\%$ in $K_L \rightarrow \pi^+ \pi^- e^+ e^-$ reported by the KTeV Collaboration [18]. However, what if two or more of the final-state particles are identical?

Consider the double-Dalitz decay of a CP-mixture (like K_L) to $e^+ e^- e^+ e^-$. (see, e.g., [19]). For low $M(e^+ e^-)$ this process is like $K_L \rightarrow \gamma\gamma$, with photons having relative linear polarizations (\parallel, \perp) for CP = $(+, -)$. Interference between CP-even and -odd decays can give a non-vanishing value of $\langle \sin\phi \cos\phi \rangle$, where ϕ is the angle between normals to the $e^+ e^-$ planes.

Now consider the case of $B \rightarrow V_1 V_2$, with each V decaying to two pseudoscalar mesons P . (For an extensive discussion of the formalism, see [20].) One extracts triple products (TPs) from angular analyses:

$$A_T \equiv \frac{\Gamma(\text{TP} > 0) - \Gamma(\text{TP} < 0)}{\Gamma(\text{TP} > 0) + \Gamma(\text{TP} < 0)}; \quad \text{TP} \equiv p_1 \cdot (p_2 \times p_3); \quad (4.1)$$

they are tiny in the SM. A true T-violation is signified by

$$\mathcal{A}_T^{\text{true}} \equiv \frac{\Gamma(\text{TP} > 0) + \bar{\Gamma}(\text{TP} > 0) - \Gamma(\text{TP} < 0) - \bar{\Gamma}(\text{TP} < 0)}{\Gamma(\text{TP} > 0) + \bar{\Gamma}(\text{TP} > 0) + \Gamma(\text{TP} < 0) + \bar{\Gamma}(\text{TP} < 0)}. \quad (4.2)$$

The matrix element for $B(p) \rightarrow V_1(k_1, \varepsilon_1) + V_2(k_2, \varepsilon_2)$ can be written

$$M = a\varepsilon_1^* \cdot \varepsilon_2^* + \frac{b}{m_B^2}(p \cdot \varepsilon_1)(p \cdot \varepsilon_2) + i\frac{c}{m_B^2}\varepsilon_{\mu\nu\rho\sigma}p^\mu q^\nu \varepsilon^{*\rho} \varepsilon^{*\sigma}; \quad q \equiv k_1 - k_2 \quad (4.3)$$

The transversity amplitudes depend on a, b, c as $A_\parallel(a)$, $A_0(a, b)$, and $A_\perp(c)$. Under CP conjugation, $a \rightarrow \bar{a}$, $b \rightarrow \bar{b}$, $ic \rightarrow -i\bar{c}$. Angular distributions depend on the angle ϕ and polar angles θ_1, θ_2 , each in the rest frame of the decaying V_1 or V_2 :

$$\begin{aligned} \frac{d\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi} &\sim |A_0|^2 \cos^2\theta_1 \cos^2\theta_2 + (1/2)|A_\perp|^2 \sin^2\theta_1 \sin^2\theta_2 \sin^2\phi \\ &+ (1/2)|A_\parallel|^2 \sin^2\theta_1 \sin^2\theta_2 \cos^2\phi + (1/2\sqrt{2})\text{Re}(A_0 A_\parallel^*) \sin 2\theta_1 \sin 2\theta_2 \cos\phi \\ &- (1/2\sqrt{2})\text{Im}(A_\perp A_0^*) \sin 2\theta_1 \sin 2\theta_2 \sin\phi - (1/2)\text{Im}(A_\perp A_\parallel^*) \sin^2\theta_1 \sin^2\theta_2 \sin 2\phi. \end{aligned} \quad (4.4)$$

The last two terms are T-odd and of two distinct types.

The interfering amplitudes are characterized by a weak phase difference ϕ_w and a strong phase difference δ . In addition to the “true” TP $\mathcal{A}_T^{\text{true}}$ defined above, one can define [4] a “fake” TP:

$$\mathcal{A}_T^{\text{fake}} = \frac{\Gamma(\text{TP} > 0) - \bar{\Gamma}(\text{TP} > 0) - \Gamma(\text{TP} < 0) + \bar{\Gamma}(\text{TP} < 0)}{\Gamma(\text{TP} > 0) + \bar{\Gamma}(\text{TP} > 0) + \Gamma(\text{TP} < 0) + \bar{\Gamma}(\text{TP} < 0)}, \quad (4.5)$$

Table 2: Longitudinal and transverse fractions f_L and f_T for some $b \rightarrow s$ -penguin $B \rightarrow VV$ processes.

| | $B_s \rightarrow \phi\phi$ | $B^+ \rightarrow \phi K^{*+}$ | $B^+ \rightarrow \rho^0 K^{*+}$ | $B^0 \rightarrow \rho^0 K^{*0}$ |
|-------|-----------------------------|-------------------------------|---------------------------------|---------------------------------|
| | [22] | [23] | [24] | [24] |
| f_L | $0.348 \pm 0.041 \pm 0.021$ | $0.49 \pm 0.05 \pm 0.03$ | $0.52 \pm 0.10 \pm 0.04$ | $0.57 \pm 0.09 \pm 0.08$ |
| f_T | $0.652 \pm 0.041 \pm 0.021$ | $0.51 \pm 0.05 \pm 0.03$ | $0.48 \pm 0.10 \pm 0.04$ | $0.43 \pm 0.09 \pm 0.08$ |

where $\text{TP}_{\text{true}} \propto \sin \phi_w \cos \delta$, $\text{TP}_{\text{fake}} \propto \cos \phi_w \sin \delta$. The two T-odd observables are

$$A_T^{(1)} \equiv \frac{\text{Im}(A_{\perp} A_0^*)}{|A_0|^2 + |A_{\parallel}|^2 + |A_{\perp}|^2}, \quad A_T^{(2)} \equiv \frac{\text{Im}(A_{\perp} A_{\parallel}^*)}{|A_0|^2 + |A_{\parallel}|^2 + |A_{\perp}|^2}. \quad (4.6)$$

For CP conjugates, one has similar definitions with barred amplitudes and a minus sign from complex conjugation of the imaginary coefficient of c . The TP asymmetries \mathcal{A}_T then satisfy

$$\mathcal{A}_T^{\text{true}} \propto \text{Im}(A_{\perp} A_i^* - \bar{A}_{\perp} \bar{A}_i^*), \quad \mathcal{A}_T^{\text{fake}} \propto \text{Im}(A_{\perp} A_i^* + \bar{A}_{\perp} \bar{A}_i^*), \quad (i = 0, \parallel). \quad (4.7)$$

The observables $A_T^{(1,2)}$ are related to those in Dorigo's talk [21] by “ u ” $\leftrightarrow A_T^{(2)}$; “ v ” $\leftrightarrow A_T^{(1)}$; he reports on their measurement in $B_s \rightarrow \phi\phi$.

The decays $B \rightarrow \phi K^*$ and $B_s \rightarrow \phi\phi$ are both dominated by the $b \rightarrow s$ penguin diagram. Factorization predicts dominant longitudinal polarization of the vector mesons, in contrast to observations [22, 23, 24] (Table 2). By contrast, the tree-dominated decay $B^0 \rightarrow \rho^+ \rho^-$ has $f_L = 0.992 \pm 0.024^{+0.026}_{-0.013}$ [25], or nearly 1 as predicted. There is no reason to trust factorization for the penguin amplitude, which may be due to rescattering from charm-anticharm intermediate states.

From $B^0 \rightarrow \phi K^{*0}$ amplitudes quoted by [4] we estimate

$$A_T^{(1)} = -0.260 \pm 0.048; \quad \bar{A}_T^{(1)} = 0.203 \pm 0.050; \quad A_T^{(2)} = 0.005 \pm 0.070; \quad \bar{A}_T^{(2)} = 0.010 \pm 0.064. \quad (4.8)$$

These values imply a large fake $A_T^{(1)}$ (since $A_T^{(1)} - \bar{A}_T^{(1)} \neq 0$); no true $A_T^{(1)}$ (since $A_T^{(1)} + \bar{A}_T^{(1)}$ is consistent with zero); and no fake *or* true $A_T^{(2)}$ (since both $A_T^{(2)}$ and $\bar{A}_T^{(2)}$ are consistent with zero). The large fake $A_T^{(1)}$ simply reflects the importance of strong final-state phases.

5. $B_s \rightarrow J/\psi\phi$ vs. $B_s \rightarrow J/\psi f_0$

Helicity or transversity analysis for $B_s \rightarrow J/\psi\phi$ (S-, P-, D-wave) is avoided for $B_s \rightarrow J/\psi f_0$ (pure P-wave). As $\text{CP}(J/\psi) = \text{CP}(f_0) = +$, the overall final state is CP odd. An estimate of the rate for this process [26] is

$$R_{f_0/\phi} \equiv \frac{\Gamma(B_s \rightarrow J/\psi f_0, f_0 \rightarrow \pi^+ \pi^-)}{\Gamma(B_s \rightarrow J/\psi \phi, \phi \rightarrow K^+ K^-)} \simeq 20\%, \quad (5.1)$$

to be compared with experimental values $0.252^{+0.046+0.027}_{-0.032-0.033}$ [27], $\simeq 0.18$ ($\sim 30\%$ stat. error) [28], and $0.292 \pm 0.020 \pm 0.017$ [21]. The CKM structure for this process is the same as for $B_s \rightarrow J/\psi\phi$. Although f_0 decays mainly to $\pi\pi$, it seems to be “fed” mainly from $s\bar{s}$: Comparing $J/\psi \rightarrow \phi\pi\pi$ and $J/\psi \rightarrow \omega\pi\pi$ [29], one sees a $\pi\pi$ peak at $M(f_0) \simeq 980$ MeV in $\phi\pi\pi$, not $\omega\pi\pi$.

6. New physics constraints

Two (of ~ 100) theoretical analyses [30, 31] emphasize the correlation between a_{sl}^q , Δm_q , $\Delta \Gamma_q$, and the mixing angle ϕ_q , where $A_{sl}^b = (0.506 \pm 0.043)a_{sl}^d + (0.494 \pm 0.043)a_{sl}^s$. The questions of whether β_s or a_{sl}^q are nonstandard are separate; they are related by $a_{sl}^q = (|\Delta \Gamma_q|/\Delta m_s) \tan \phi_q$. If the D0 dimuon asymmetry is mainly from a_{sl}^s , Ref. [31] finds $a_{sl}^s = (-12.5 \pm 4.8) \times 10^{-3}$ by combining with the D0 measurement $(-1.7 \pm 9.1) \times 10^{-3}$. Using in this formula the (CDF, LHCb) average $\Delta m_s = (17.70 \pm 0.08) \text{ ps}^{-1}$ and the (CDF, D0) average $\Delta \Gamma_s = 0.094 \pm 0.031 \text{ ps}^{-1}$, one expects $\phi_s = (-67_{-7}^{+18})^\circ$. Comparing with $\phi_M^s = (-39 \pm 17)^\circ$, this would favor slightly larger $\Delta \Gamma_s$ or a nonstandard value of a_{sl}^d . In Ref. [5] it is noted that one must respect the SM prediction of Δm_q . New physics must affect mainly *phases* of mixing amplitudes.

7. A cursory look at new physics scenarios

Supersymmetry has generic flavor-changing (but controllable) effects [32]. Randall-Sundrum [33] scenarios in which different quarks lie at different points along a fifth dimension offer a language for understanding quark mixings; but there is no predictive scheme yet. Theories with an extra (flavor-changing) Z can induce mixing as desired. In Ref. [31] a contribution to $\Delta \Gamma$ is introduced through a new light pseudoscalar (an on-shell state in $B_s \leftrightarrow \bar{B}_s$). These are just some examples of a wealth of models on the market. Some of them predict other observable consequences but there are too many to enumerate exhaustively. Two of my current favorites are (1) a fourth generation, and (2) a hidden sector.

Lunghi and Soni [34] note the tension between $\sin 2\beta = \sin 2\phi_3 = 0.668 \pm 0.023$ (measured in B decays) and that (0.867 ± 0.048) in (their) CKM fit. They note effects of new physics on both $\Delta \text{Flavor} = 1$ (penguin) and $\Delta \text{Flavor} = 2$ (box) amplitudes but give no specifics on β_s or a_{sl}^s .

In a “hidden sector” let an extended gauge sector G describe dark matter, and let there be particles Y with charges in both the SM and in G , and particles X with charges only in G . A box diagram describing $B_s - \bar{B}_s$ mixing in this scenario is shown in Fig. 3. Table 3 gives examples of ordinary, mixed, and “shadow” matter. There are clearly many opportunities in such a scenario for new contributions to penguin and box diagrams.

Table 3: Types of matter and their SM and hidden charges.

| Type of matter | Std. Model | G | Example(s) |
|----------------|------------|-----------|-----------------------------|
| Ordinary | Charged | Uncharged | Quarks, leptons |
| Mixed (Y) | Charged | Charged | Superpartners |
| Shadow (X) | Uncharged | Charged | E'_8 of $E_8 \otimes E_8$ |

8. Summary

B_s decays and mixing provide potential mirrors of new physics. While the phase β_s has moved toward its Standard Model value, even the currently measured value of β_s should be manifested in

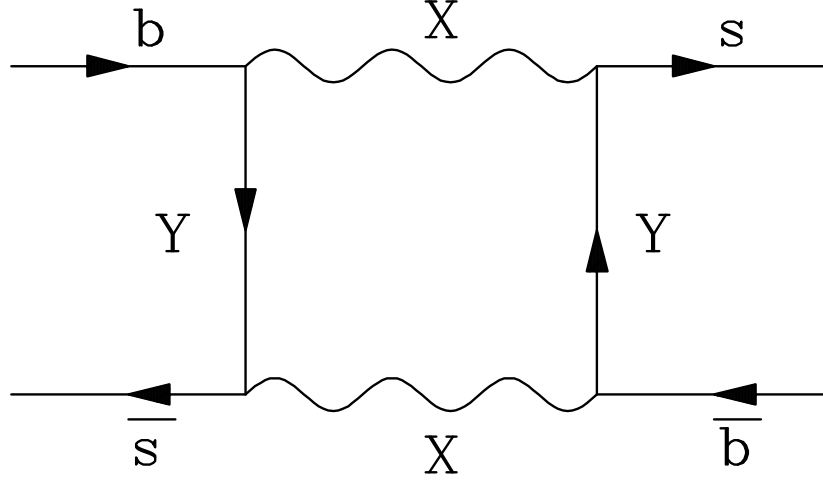


Figure 3: Diagram utilizing a hidden sector describing B_s - \bar{B}_s mixing.

time-dependent quantities.

The D0 collaboration [2] claims a dimuon charge asymmetry. At this conference [15] CDF has reported a remeasurement of $\bar{\chi}$ and we look forward to their further progress on dimuons. The signal requires subtraction of a big kaon background. Is what's left really due to b quark decays? We have proposed an impact parameter cut of $b < 100 \mu\text{m}$ to find out [3].

Using triple products in four-body decays, one can construct T-odd observables providing strong and weak phase information. There is interest in what new physics one can learn from $B_s \rightarrow \phi\phi$ [21].

As for whether there is new physics in any of the above hints, I urge you to have your favorite model ready; there are enough to go around.

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